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Natural resources and environmental economics, TSE

Tutorial 5a - Exhausible Resources - Market Economies

Exercise 1: A Monopoly of an exhaustible resource

Now we turn to the case where a monopolist exploits an exhaustible natural resource. We start by assuming that extraction is costless ($C(s_t) = 0$). The inverse demand function is given by some function $p_t(s_t)$.

a) State the general problem of the monopolist. Write the Hamiltonian and solve the problem using the definition of the inverse elasticity of demand $\epsilon_{p,s}$.

b) Now assume that final demand is given by the isoelastic inverse demand function $p_t(s_t) = b \cdot s_t^{-\alpha}$. Compute the first order condition for this case and compare the result with the competitive case and the optimal extraction.

c) Another frequently used demand function is the linear demand case where $p_t(s_t) = a - bs_t$. Comment on the variation of the demand elasticity. What are plausible arguments to support this shape of the demand elasticity? Solve the model for this case and compare the result with b).

[**Optimal Control Exercises**]

Exercise 2: Extraction in the competitive case

In the last problem set, we derived the Hotelling rule from the social planner's objective. Now, we look at the case where the resource is sold through a competitive market, i.e., firms and consumers are pricetakers. Recall that in the competitive case, maximizing profits is equivalent to maximizing social surplus. Consumer surplus or utility measured as the integral under the inverse demand curve can be written as $u(s_t) = \int_0^{s_t} p(x)dx$. We abstract from extraction costs.

a) State the optimization problem and derive the Hamiltonian.

b) Compute the first order conditions (Note that the derivative $d\left(\int_0^{s_t} p(x)dx\right)/ds = p(s_t)$).

c) Derive from the two conditions the Hotelling rule $\dot{p}/p = r$. Note that the Hotelling rule holds for the competitive industry case if the interest rate r equals the social rate of time preference ρ.

Exercise 3: A simple optimal growth model

Consider an economy where all variables are expressed in per capita terms. Output is produced from capital k (per capita) solely as $y = k^\alpha$ (Cobb Douglas). Instantaneous utility is assumed to be logaritmic depending only upon consumption. $u(c_t) = \ln c_t$. As output can either be consumed or invested, the change of the capital stock $\dot{k}$ equals the share of output which is not consumed (no depreciation). Hence, $\dot{k} = y - c$.

a) State the social planner's maximization problem as an optimal control program. Which are the state and control variables?

b) Write the Hamiltonian and derive the first order conditions. Show that the necessary conditions for an optimal growth path can be written as $\frac{\dot{c}}{c} = \alpha k^{\alpha-1} - \rho$. (Hint: log-differentiate the first condition you get and solve both FOCs for $\frac{\dot{\lambda}}{\lambda}$).