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Tutorial 4 - Pigouvian Taxes and Pollution Permits II

Corrections

- **Q 1:** *Write the environmental agency problem as a constrained minimization problem (Hint: take care of the activity constraint $q_i \leq \bar{q}_i$!). Using the trick:* $\min f(x) \equiv \max -f(x)$ *transform this problem into a constrained maximization problem. Write down the corresponding Lagrangian and compute the first order conditions for optimality of an individual norms vector $\{q_1, \cdots, q_i, \cdots, q_I\}$ satisfying the global objective $Q \leq \bar{Q}$. Denote by λ the Lagrange multiplier associated to the global pollution constraint and by α_i the Lagrange multiplier associated to the pollution abatement activity constraint for each firm i.*

  The corresponding maximization problem is:

$$Max_{\{q_i|i=1,\cdots,I\}} \quad -\sum_{i=1}^{I} C_i(q_i)$$
$$s.t. \quad \sum_{i=1}^{I} q_i \leq \bar{Q}$$
$$q_i \leq \bar{q}_i$$

  Note that the positivity constraint over q_i can be neglected thanks to the assumption $\lim_{q_i \downarrow 0} c_i(q_i) = -\infty$ which exclude that the firms try to eliminate completely the pollution they create. The corresponding Lagrangian is:

$$\mathcal{L} = -\sum_{i=1}^{I} C_i(q_i) + \lambda(\bar{Q} - \sum_{i=1}^{I} q_i) + \sum_{i=1}^{I} \alpha_i(\bar{q}_i - q_i)$$

  The first order conditions are:

$$\mathcal{L}_{q_i} = 0 \implies -c_i(q_i) = \lambda + \alpha_i \qquad i = 1, \cdots, I$$
$$\alpha_i \geq 0, \alpha_i(\bar{q}_i - q_i) = 0 \quad i = 1, \cdots, I$$

It results from these conditions that all environmentally active firms (that is firms setting a pollution level equal to $q_i < \bar{q}_i$) should equalize their marginal pollution abatement cost to λ.

- **Q 2:** *Depending upon the severity of the pollution constraint $\bar{Q}$, does all firms have to make a pollution reduction effort ? Explain.*

  If for a given firm $\lambda < -c_i(\bar{q}_i) = -\underline{c}_i$ then this firm should not undertake any pollution reduction effort. Thus typically for a given norm $\bar{Q}$, one should conclude that some firms should not reduce their pollution. This may be seen as a consequence of two facts: either the firm has a low pollution level without regulation ($\bar{q}_i$ is "low" for this firm with respect to the others) and the firm is efficient in pollution reduction (its marginal cost curve is low) or either the firm pollutes a lot without regulation but its marginal cost of pollution abatement is high with respect to the other firms. The following graph illustrates this conclusion. For the level of λ which corresponds to $\bar{Q}$, the low cost firm 1 and the high cost firm 4 should not do anything. Only the firms 2 and 3 reduce their pollution and thus equalize their marginal cost of abatement to λ.

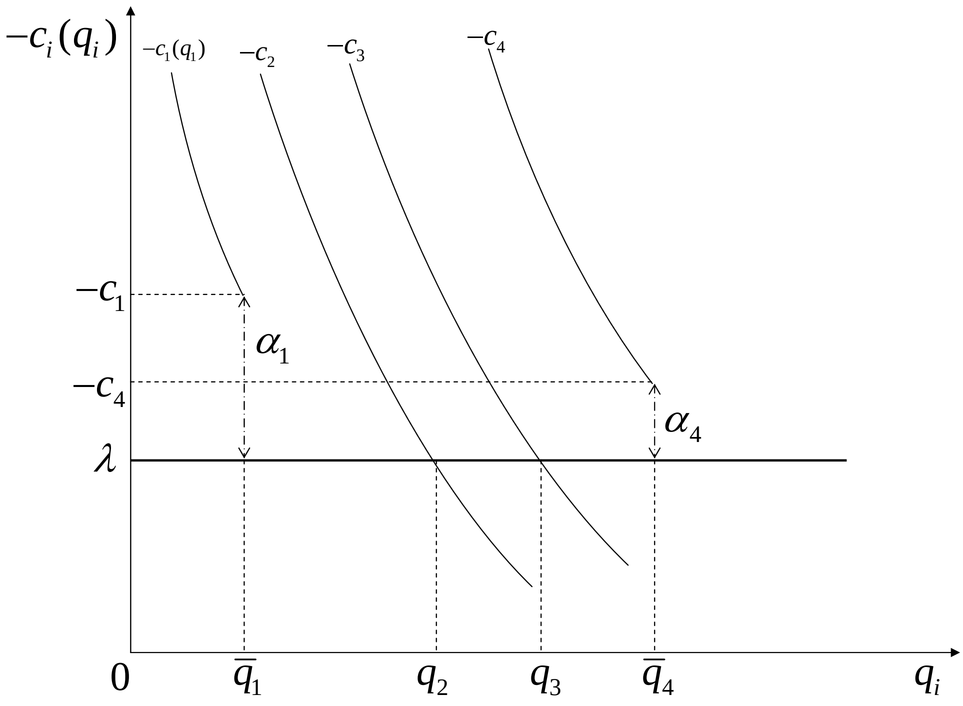

Figure 1: **Cost minimizing pollution efforts.**

Additional Comment : Usually, the problem of determining a socially efficient pollution level is cast in a cost-benefit analysis framework. The agency has to balance between an environmental benefit function (or an environmental damage function) and the cost of a pollution reduction. The result will be the equalization between the marginal environmental benefit (or the marginal environmental damage decrease) and the marginal cost of such a pollution reduction. This is the kind of result one finds into any environmental economics textbook (see Kolstad for example). But what is the "marginal cost of pollution abatement" for a population of polluters with different abatement cost functions ? The objective of the exercise is to shed light on this point.

To achieve that, instead of performing a benefit-cost analysis I cast the problem in a cost-efficiency framework. That is, for a given environmental objective (here reach the global level $\bar{Q}$) the agency has to find the least cost way to achieve the objective. This is a quite natural requirement for an agency policy. This kind of framework is now mandated into many environmental regulations. For example, the European Water Directive imposes to the Water Agencies to design their policies in a cost-effective way, that is minimize the economic cost of reaching the water quality objectives set up in the Directive. One finds the same kind of requirement into the REACH Directive for chemicals control.

- **Q 3:** *Let K be the subset of firms in $\{1, \cdots, I\}$ being active in reducing their pollution and $\bar{K} \equiv \{1, \cdots, I\} \setminus K$ be the subset of inactive firms in pollution abatement. What do you observe for the marginal cost of pollution abatement of the active firms in K? Give an economic interpretation of the result.*

  As already observed, the firms in K which reduce their pollution level have to do so at the same marginal cost level. Furthermore this common level of the marginal cost has to be equal to the marginal opportunity cost of the global pollution constraint λ. Remember that in a maximization problem, the Lagrange multiplier associated to a given constraint measures the marginal variation of the objective function when the constraint is slightly relaxed. That is, if we denote : $C(\bar{Q}) \equiv \max(-\sum C_i(q_i)) | \sum q_i = \bar{Q}$, then $\lambda = dC(\bar{Q})/d\bar{Q}$. Now if two firms doing pollution abatement efforts bear different marginal costs, by transferring some pollution reduction effort form the most costly firm to the less costly one, it is possible to reduce the sum of the costs. Hence at the total minimum cost level, the marginal costs of operation of the firms doing an effort have to be the same.

- **Q 4:** *Suppose that the agency decides to implement a stricter global pollution norm $\bar{Q} < \bar{Q}$. What will be the consequences over:*

  - The level of the marginal abatement cost of the active firms?

  - The number of environmentally active firms?

  - The level of λ, the opportunity cost of the global pollution norm?

  With a stricter global norm, the pollution abatement activity of the firms has to be increased, hence q_i has to decrease. Since $c_i(q_i)$ has been assumed to be an increasing function of q_i, $-c_i(q_i)$ has to increase for q_i to decrease also. This requires to increase the level of λ. Thus λ has to increase when $\bar{Q}$ is decreased to $\bar{Q}'$. This implies that the marginal cost abatement of the firms increase (they make more costly efforts to reduce their pollution). Since λ increases, some firms which were not active in the original situation will now have also to reduce their pollution. A look at the graph shows that if λ increases sufficiently, the firm 4 will have to enter an active pollution reduction policy. If λ increases a lot all firms will be active.

  Additional Comment : As you can see, it is often very useful to use a graph to solve an economic exercise. I advice you strongly to do this kind of job. In an exam paper, I accept without problems pure graphical reasoning without calculations if the graph is correct and the reasoning makes sense.

- **Q 5:** *It appears that the global pollution level Q can be expressed as a function of λ, the marginal opportunity cost of the pollution norm. Let $Q(\lambda)$ be that function. Use the assumption $C.1$ and the first order conditions to deduce from the answer to Question Q.3 the sign of $dQ(\lambda)/d\lambda$. Infer from that a way to compute the optimal level of λ. Verify that this computation allows to determine the vector of the optimal individual norms $\{q_1^*, \cdots, q_i, \cdots, q_I^i\}$ to be imposed upon the different firms.*

  This was the most technical question of the exercise. The problem is the following. The first order conditions show that the marginal costs have to be equalized to λ at the optimum for firms reducing their pollution level. But these conditions provide no direct way to compute what should be the optimal level of λ and hence the optimal level of the marginal cost. Moreover we have also to determine the firms which should be active and the firms which should do nothing to reduce their pollution. Accomplish this task is called in the literature: 'deriving a closed form solution' of the model. In other words determine the optimal level of the variables as functions of the model parameters. This is a plain (and often tedious) mathematical exercise. The question 5 asked you to design a method to derive a closed form solution for the problem at hand.

For the active firms in K, the first order conditions give: $\alpha_i = 0$, $i \in K$ and thus $-c_i(q_i) = \lambda$ for $i \in K$. This last equation defines implicitly q_i as a function of λ. If $\lambda < -\underline{c}_i$, then we know that $q_i = \bar{q}_i$, and the firm is in the set $\bar{K}$. Thus the function $q_i(\lambda)$ is only defined for $\lambda \in [-\underline{c}_i, \infty)$. Furthermore since by assumption $c_i'(q_i) > 0$, then differentiating the first order condition with respect to q_i and λ:

$$-c_i'(q_i)dq_i = d\lambda \implies \frac{dq_i(\lambda)}{d\lambda} = -\frac{1}{c_i'(q_i)} < 0 \quad \lambda \in [-\underline{c}_i, \infty)$$

Hence we conclude that $dq_i/d\lambda < 0$, the pollution level of the active firms is a decreasing function of λ. This confirms the graph conclusion. Next the total pollution level Q is equal to:

$$Q = \sum_{i \in K} q_i(\lambda) + \sum_{j \in \bar{K}} \bar{q}_j$$

Let k and $\bar{k}$ be the cardinals of the sets K and $\bar{K}$, that is the numbers of firms in K and $\bar{K}$ respectively. If λ increases, α_j decreases for the firms in $\bar{K}$. For a sufficient increase of λ we can thus conclude that some firms will move from $\bar{K}$ to K. In other words, as a function of λ, $k(\lambda)$ increases with λ while $\bar{k}(\lambda)$ decreases when λ increases. Since we have shown previously that $q_i(\lambda)$ is a decreasing function of λ and since $q_i \leq \bar{q}_i$, we can conclude that, as a function of λ, $Q(\lambda)$ decreases with λ. The minimum value of λ is given by $\min -\underline{c}_i \equiv \underline{\lambda}$. For $\lambda \leq \underline{\lambda}$, no firm would make any effort. Hence, $Q(\underline{\lambda}) = \sum_i^I \bar{q}_i \equiv \tilde{Q}$. For λ increasing up to infinity, the firms do more an more efforts and the pollution level converges down to zero, thus: $\lim_{\lambda \uparrow \infty} Q(\lambda) = 0$.

Having set the limits, we know now that the global pollution level decreases between $\tilde{Q}$ and 0 for λ varying between $\underline{\lambda}$ and infinity. If $\bar{Q} > \tilde{Q}$ then the pollution constraint is not stringent: by polluting at the maximum level, the firms would create a total pollution amount lower than the pollution objective of the agency. One can set λ to zero in such a case and the optimal solution will be to let the firms pollute at their maximum level. If $\bar{Q} < \tilde{Q}$, then the equation $Q(\lambda) = \bar{Q}$ admits a unique solution λ^* in the admissible domain $[\underline{\lambda}, \infty)$. To this unique solution is associated a unique vector of individual pollution levels for the firms $(q_1^*, \cdots, q_i^*, \cdots, q_I^*)$ such that:

$$q_i^* = \begin{cases} q_i(\lambda^*) & \text{if} \quad \lambda^* > -\underline{c}_i \\ \\ \bar{q}_i & \text{if not} \end{cases} \qquad i \in \{1, \cdots, I\}$$

- **Q 6:** *Instead of setting individual norms, the environmental agency decides to put in place a pollution tax. For the given level of pollution objective $\bar{Q}$ what should be the level of the tax which would minimize the cost of compliance for the firms population? Do all firms reduce their pollution under the tax scheme? Explain.*

  Maximizing their profits, the firms should try to minimize $C_i(q_i) + tq_i$, where t is the level of the pollution tax implemented by the environmental agency, within the domain $[0, \bar{q}_i]$. Hence, either there exists $q_i < \bar{q}_i$ solution of $c_i(q) + t = 0$, either the firm prefers to pay the maximum tax $t\bar{q}_i$ and make no effort of pollution control. By setting the tax level $t = \lambda^*$, we see that the environmental agency implements its pollution objective at the minimum cost for the firms population, since $-c_i(q_i) = t = \lambda^*$. Of course, as seen above for the optimum, not all firms will reduce their pollution. For some of them the tax burden will be lower than undertaking even a minimal level of efforts.

- **Q 7:** *Last, the agency, wondering about the firms heterogeneity, decides to put in place a pollution permits market system. The agency issues a number $\bar{Q}$ of permits and sell them to the firm through an auction process without free allocation of some permits. Then the firms are free to trade the permits on the market. What will be the level of the equilibrium price for the permits on the market?*

  Firms now try to minimize $C_i(q_i) + pq_i$, p being the price of a one unit pollution permit on the permits market. If the current price of the permit $p < -\underline{c}_i$, then the firm i prefers to buy a number $\bar{q}_i$ permits at this price and make no effort of pollution control. Firms such that their minimal marginal cost of pollution reduction is lower than the price of the permits will reduce their pollution under $\bar{q}_i$. Minimizing cost firms will demand a number q_i of permits solution of $-c_i(q_i) = p$

resulting in an individual demand function of permits $q_i(p)$. Under our assumptions, $q_i(p)$ will be a decreasing function of p. When the price of the permits goes up, the firms prefer to do more in house pollution abatement rather than pay more for being allowed to pollute. Let $Q(p)$ be the aggregate demand for pollution permits:

$$Q(p) \quad = \quad \sum_{i \in K(p)} q_i(p) + \sum_{j \in \bar{K}(p)} \bar{q}_j$$

Since at the equilibrium, the permits market must clear, $Q(p) = \bar{Q}$. Since we have seen before that individual demands functions are either decreasing with p or inelastic to p if $p < -\underline{c}_i$, we conclude that there exists a unique equilibrium level of p. Remembering questions 5 and 6, it appears immediately that $p = \lambda^* = t$, that is the equilibrium price over the permits market should be same as the optimal level of the tax in question 6 and also the same as the optimal level of the opportunity cost of the norm constraint in question 5. This conclusion comes at no surprise. Since there is perfect information, no transaction costs and no strategic behavior of the firms, all policy tools will be able to implement the optimum, either under the form of a vector of individual norms, a pollution tax or an equilibrium price over a pollution permits market.

- **Q 8:** *If the agency decides to reduce the pollution allowance from $\bar{Q}$ to $\bar{Q}' < \bar{Q}$ by buying back on the market the corresponding amount of permits, do all firm willing to sell back some fraction of their permits? Explain.*

  Of course not. A buy back of permits by the agency will induce an equilibrium price increase. But for a moderate price increase, there will remain firms which would prefer to hold a $\bar{q}_i$ number of permits and no do an in house pollution reduction effort. These firms will not want to sell their permits to the agency. Note that active firms will participate in the buying back operation, finding better to sell permits to the agency while doing more in house pollution abatement.

- **Q 8:** *Without loss of generality, rank the firms by increasing order of pollution levels without abatement effort, so that $\bar{q}_1 < \bar{q}_2 < \cdots < \bar{q}_i < \cdots, \bar{q}_I$. In order to circumvent the lobbying activity of the firms, the agency decides to grant a free pollution allowance to a subset J of the most polluting firms in case of no regulation, that is they can pollute as* *much as they want without having to buy permits. The agency wants to achieve the same global pollution objective. What will be the effect of such a free allowance to pollute over the equilibrium price of the permits?*

Let $\bar{q}_J$ be the lower bound of the free pollution levels in J, that is under our ranking condition : $J = \{i|\bar{q}_J \leq \bar{q}_i \leq \bar{q}_I\}$. Denote by j the firm at the lower bound, that is $\bar{q}_j = \bar{q}_J$. Then two possibilities may arise. If $p < -\underline{c}_j$ the firms in J benefiting from an exemption did not make previously any effort to reduce their pollution. Their behavior will be the same after the free granting of permits. To implement its objective the agency should torn down a number of permits equal to $\sum_J \bar{q}_i$, the amount previously purchased by the firms in J. This will result into the same decisions for the other firms and the equilibrium price will remain the same. The only consequence will be windfall profits for the firms in J in terms of pollution permits costs and a corresponding loss of revenues from permits sales for the agency.

If $p > -\underline{c}_j$, some firms in J were making pollution reduction efforts in the original situation. With a free allowance to pollute, these firms stops doing pollution reduction and thus the total pollution increases. By cutting back an equivalent number of permits, the agency makes increase the price of the permits on the market. The firms outside the exemption scheme will have to do more in house pollution abatement efforts for the agency to comply with its environmental objective. This was not the case in the example studied in class since all polluters should continue to buy permits even with a free granting of permits initially. The fact that in this model, some firms can evade completely the regulation scheme results in a transfer of the burden of the pollution control policy over the remaining firms.