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Tutorial 3 – Water pollution (based on the final exam 09/10) CORRECTION

Consider I cities located around a lake. Each city pumps water inside the lake to satisfy the water demand by urban users. The pumping costs are assumed to be null. Let x_i be the water consumed by a city i. By consuming the water the urban users pollute it. Let q_i be the pollution rate generated by a city i in solid waste equivalent. Used water is collected by each city and sent to a sanitation plant for water clean up. Let a_i be the amount of pollution eliminated by the sanitation plant of a city i and let $z_i \equiv q_i - a_i$ be the net pollution rate. Net pollution is then discharged into the lake. Eliminating pollution incurs an operating cost of the sanitation plant $C(a_i)$, where $C(0) = 0$, $c(a_i) \equiv dC(a_i)/da_i > 0$, $c(0) = 0$ and $c'(a_i) > 0$ with $\lim_{a_i \uparrow q_i} c(a_i) = +\infty$. This last assumption ensures that no city will eliminate completely its pollution and so $z_i > 0$ for all the cities. Note also that all cities use the same sanitation technology and hence share the same cost function.

The urban users can make efforts to control their gross pollution rate q_i but these efforts are costly. Thus the gross surplus function of urban users of city i may be defined as $V(x_i, q_i)$ where $v^x(x_i, q_i) \equiv \partial V(x_i, q_i)/\partial x_i > 0$ if $x_i < \bar{x}_i(q_i)$ and equal 0 if $x_i = \bar{x}_i(q_i)$, while $v^q(x_i, q_i) \equiv \partial V(x_i, q_i)/\partial q_i > 0$ and equal 0 if $q_i = \bar{q}_i(x_i)$. Note that all users in all cities share the same surplus function. Moreover the function V verifies:

- $\lim_{x_i \downarrow 0} v^x(x_i, q_i) = +\infty$ and $\lim_{q_i \downarrow 0} v^q(x_i, q_i) = +\infty$. Under these assumptions both $x_i > 0$ and $q_i > 0$.
- $\partial v^x/\partial x_i < 0$, $\partial v^q/\partial q_i < 0$ and $\partial v^x/\partial q_i = \partial v^q/\partial x_i > 0$. Under these assumptions the marginal surpluses are strictly decreasing functions of their arguments.
- Denoting by $V_{xx} < 0$, $V_{qq} < 0$ and $V_{xq} > 0$ the above second order partial derivatives, $V_{xx}V_{qq} - (V_{xq})^2 > 0$. Under this assumption, the function $V(x_i, q_i)$ is concave in (x_i, q_i).

Let $Z \equiv \sum_{i=1}^{I} z_i$ be the aggregate pollution discharged into the lake by the cities. This pollution creates an environmental damage for the urban populations living around the lake. Let $D(Z)$ be this damage in welfare terms. Assume that $D(0) = 0$, $D'(Z) > 0$ and $D''(Z) > 0$: the damage is an increasing and convex function of the lake pollution rate.

Q1: *Check that the urban users maximizing their surplus should choose the water amount $\bar{x}(q_i)$ which nullify their marginal surplus v^x. Define by $W(q_i) \equiv V(\bar{x}(q_i), q_i)$ their optimized surplus with respect to x_i and define accordingly $w(q_i) \equiv dW(q_i)/dq_i$. Check first that $w(q_i) = V^q$ (Hint : think of the envelope theorem since $v^x(\bar{x}, q_i) = 0$). Check then that under the assumptions given above $w'(q_i) < 0$ (Hint : you should get first the expression of $d\bar{x}/dq_i$ using the fact that $V^x(\bar{x}, q_i) = 0$ and then make use of the concavity of the function V embodied inside the item 3 of the assumptions).*

Answer: Since water delivery bears no costs, the users will maximize their surplus from water consumption, that is equalize to zero their marginal surplus in this dimension. Thus $x_i = \bar{x}(q_i)$. Since $W(q_i) = V(\bar{x}(q_i), q_i)$ we get $w(q_i)$ by differentiating W with respect to q_i. This results in:

$$w(q_i) = \frac{d\bar{x}(q_i)}{dq_i} V_x(\bar{x}(q_i), q_i) + V_q$$

But since $V_x(\bar{x}(q_i), q_i) = 0$ by definition of $\bar{x}(q_i)$, this results in $w(q_i) = V_q$ as asked. Next differentiating once again with respect to q_i:

$$w'(q_i) = V_{qx} \frac{d\bar{x}(q_i)}{dq_i} + V_{qq}$$

Furthermore:

$$V_x(\bar{x}(q_i), q_i) = 0 \implies V_{xx} \frac{d\bar{x}(q_i)}{dq_i} + V_{xq} = 0 \implies \frac{d\bar{x}(q_i)}{dq_i} = -\frac{V_{xq}}{V_{xx}}$$

Inserting this expression of $d\bar{x}(q_i)/dq_i$ inside the previous relation, one gets:

$$w'(q_i) = -\frac{V_{xq}}{V_{xx}} V_{qx} + V_{qq} = \frac{1}{V_{xx}} \left[V_{xx} V_{qq} - (V_{qx})^2 \right] < 0$$

since by assumption $V_{xx} < 0$ and $V_{xx}V_{qq} - (V_{qx})^2 > 0$.

Q 2 : *The cities agree to delegate the management of the lake pollution to a central water agency. The objective of this authority is to maximize the net surplus of the population living around the lake taking into account the environmental damages. Write down the corresponding maximization problem in terms of the function $W(q_i)$ for the cities. Give the first order conditions for maximization with respect to q^i and a_i. Make an economic comment about these conditions. Check that all cities having identical cost and the same surplus functions, they should pollute the same and make the same pollution abatement effort. You will denote by q and a these common levels and by $Q \equiv Iq$ and $A \equiv Ia$ the corresponding aggregate levels. Show on a graph in the (a, q) plane how the optimal levels of a and q can be determined (Hint : differentiate completely the first order conditions to get two relations linking q to a, $q^q(a)$ and $q^a(a)$ and check that they are increasing functions of a.*

Answer : The agency seeks to maximize the net total surplus that is solve:

$$\max_{(q_i, a_i)} \quad \sum_{i=1}^{I} [W(q_i) - C(a_i)] - D(\sum_{i=1}^{I} (q_i - a_i))$$

The optimal pair (q_i, a_i) must be the solution of the system of first order conditions:

$$\begin{aligned} w(q_i) &= D'(z) \\ c(a_i) &= D'(z) \end{aligned}$$

Thus the optimal management requires that: $w(q_i) = c(a_i) = D'(z)$. The marginal surplus from polluting more $w(q_i)$ has to be equalized both to the marginal cost of eliminating pollution $c(a_i)$ and to the marginal environmental damage $D'(z)$. This is a basic result in environmental economics. Since the cities are perfectly identical, they should pollute the same and make the same pollution abatement effort.

In the symmetric case, $z = I(q-a)$ and the first order conditions deliver two implicit relationships between q and a:

$$\begin{aligned} w(q) = D'(I(q-a)) &\implies q \equiv q^q(a) \\ c(a) = D'(I(q-a)) &\implies q \equiv q^a(a) \end{aligned}$$

Performing a complete differentiation of the first order condition with respect to q and a, one obtains:

$$\begin{aligned} w'(q)dq &= D''(z)I(dq - da) \implies \frac{dq^q(a)}{da} = -\frac{ID''(z)}{w'(q) - ID''(z)} > 0 \\ c'(a)da &= D''(z)I(dq - da) \implies \frac{dq^a(a)}{da} = \frac{c'(a) + ID''(z)}{ID''(z)} > 0 \end{aligned}$$

since $D''(z) > 0$ and $c'(a) > 0$ by assumption and it has been shown that $w'(q) < 0$. Thus $q^q(a)$ and $q^a(a)$ describe two increasing curves in the (a, q) plane. Provided that these curves cross themselves, they define the optimal pair (q, a).

Q 3 : *The central agency is not in position of imposing to the urban users to reduce their own emissions and it can act only upon a_i the abatement efforts of the cities. What can be said about the environmental effect of such a regulation with respect to a Pareto optimal situation?*

In the normal case, Pareto optimality requires that the consumers must refrain from polluting, that is pollute less than $\bar{q}(\bar{x})$. If the regulator cannot act upon q the residents of the cities will pollute at the maximum rate $\bar{q}(\bar{x})$. If the regulator wants to reach the optimal level of pollution, this means that it should ask to the cities to abate more pollution, that is increase their effort a. Since the marginal cost $c(a)$ will increase above its optimum level while $w(q)$ will decrease below its optimum level (remember that $w'(q) < 0$) the Pareto conditions could not be met.

Q 4 : *Without regulation, $a_i = 0$ and $q_i = \bar{q}_i(\bar{x}_i)$, the cities would make no abatement effort and the users would pollute at the maximum rate. The water agency decides to implement the optimal pollution level by setting a tax depending upon z_i the net pollution level of a city i. This tax is levied upon each city. Remember that cities being identical, the tax rate will be the same for all cities. The city administration is then free to impose a charge upon pollution emissions to the users. Compute the level of the optimal tax and the tariff rate upon q_i the city should impose.*

Answer : Since there is no charge upon water consumption, the users continue to consume the maximum amount $\bar{x}(q)$. But they will have to pay for pollution emitted while the city must get the correct incentive to reduce pollution. The cities being symmetric, they will adopt the same taxation policy. This will result into the same q_i, a_i and z_i, hence we have $z = I(q - a)$ as before. Let p be the tax rate imposed by the agency, then a city is faced with the following problem:

$$Max \qquad W(q) - C(a) - pz = W(q) - C(a) - p(q - a)$$

If the agency sets $p = D'(z)$, it is immediately checked that the city will choose (q, a) solution of $w(q) = c(a) = p = D'(z)$, that is implement the optimal policy in terms of pollution allowance and pollution abatement. The city administration covers its marginal cost. Assume that the city is composed of N identical residents with utility functions $U(q_j)$ where j indexes the residents. Let $u(q_j) \equiv dU/dq_j$ the marginal utility. Then $W(q) \equiv \sum_{j=1}^{N} U(q_j) = NU(q/N)$ since the residents being the same they behave the same. The city administration imposes a tariff upon pollution τ per unit sent to the sanitation plant. The residents maximize their net surplus $U(q_j) - \tau q_j$. This results into a pollution choice q_j such that $U'(q_j) \equiv u(q_j) = \tau$. Since $w(q) \equiv W'(q) = U'(q/N) = u(q/N) = \tau$, we see that the city administration has to set $\tau = p$ in order to decentralize the optimal level of pollution z.

Note that the budget balance of the city administration is thus: $\tau q - C(a) - p(q - a) = pa - C(a)$. A priori, it is thus not clear whether the city's budget is balanced, negative or positive. In any case, the surplus or deficit should be redistributed/taxed according to lump sum transfers to the citizens in order to not avoid distortions. (This was not asked in the question).

Q 5 : *Being unable to observe the individual pollution emissions, the cities have to set a fee upon water consumption x_i in the city. Compute the optimal level of such a fee. Compare to the previous question results.*

Answer : This question was a bit more intricate. The idea is that by taxing water, the city has the opportunity to shape their citizens' behaviour regarding pollution (Second-Best Policy). However, this also gives an incentive to the residents to pollute more to compensate their loss on the water side. The problem for the residents is now:

$$\max \qquad V(\bar{q}(x), x) - wx$$

where w denotes the water tax rate. Since there is no direct regulation of gross pollution emissions from the residents, they pollute at the maximum amount that is $\bar{q}(x)$ solution of $V_q(q, x) = 0$. Applying once again the envelope theorem, it results that the optimal water consumption level under tax should be solution of $V_x(\bar{q}(x), x) \equiv W'(x) = w$. Since the surplus is symmetric in x and q, $W'(x)$ is a decreasing function of x. Thus this equation defines $x(w)$ the residents' water demand. The city problem may thus be shaped as:

$$Max \qquad W(x) - C(a) - pI(\bar{q}(x) - a)$$

leading to the first order condition: $W'(x) = pId\bar{q}(x)/dx$ while $c(a) = p$ as before. Since the agency should set $p = D'(z)$ to reach the optimal pollution level in the lake, w has to be set at the level solution of: $w = W'(x) = D'(z)d\bar{q}(x)/dx$.

Intuition: The water tax rate in this setting should be set such that it equals the marginal effect on the damage due to the effect on the satiation level $D'(z)d\bar{q}(x)/dx$. When loking at the welfare implications, the first-best solution above is clearly preferrable over the second-best policy and the regulation of this question through a water tax rate requires to put a higher welfare cost upon the behavior of the residents, resulting in a lower utility level than in the previous case.